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Teaching optic laboratory: Understanding the reflection and transmission of polarized light – Fresnel's equations – Brewster's angle and index of refraction of two transparent materials

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ABSTRACT

We present a work of the laboratory procedures to estimate the index of refraction, n, of two common transparent materials, glass and Plexiglas, through the understanding of the reflection and transmission of polarized light – Fresnel's equations and Brewster's angle. We intend to show how the refracted light intensity is affected by the incident angle and the plane of polarization of the incident light between dielectric media, air- glass and air-Plexiglas. The Fresnel's equations were verified, and the Brewster's angle was determined for the used materials from the data plot of reflectance against incident angle of light polarized parallel and perpendicular to the plane of interface through glass and plexiglass plates, respectively. The experimental value of Brewster's angle for glass and Plexiglas was (57±0.5) ° and (56±0.5) °, and the calculated average value of refractive index was n_{glass} = 1.539±0.006 and n_{plex} = 1.482±0.008 compared to the accepted value of n_{glass} = 1.52 and n_{plex}=1.495, respectively. The conducted experiment was accurate as the value of experimental refractive index was in good agreement with the accepted value.

1. Introduction

A light wave is an electromagnetic (EM) wave. The EM waves are transverse waves consisting of varying electric and magnetic fields that oscillates perpendicular to the direction of propagation.

For an electromagnetic (EM) wave, we define the direction of polarization to be the direction parallel to the electric field. The polarization is characterized by its wave's oscillation relative to the direction of propagation [Edmund Optics, Semrock]

a) Oscillations of the wave in a vertical plane are said to be vertically polarized

b) Oscillations of the wave in horizontal plane are said to be horizontally polarized

Unpolarized light can be represented by random combination of parallel and perpendicular polarized light to the plane of incidence. An ideal polarizer is a material that passes only EM waves for which the electric field vector is parallel to its transmission axis. The electric field is a vector that can be written in terms of the components parallel and perpendicular to the polarizer's transmission axis.

If natural (unpolarized) light is incident on a dielectric surface, the Fresnel equations can be used to describe the reflection and transmission coefficients of an incoming electromagnetic plane wave on the interface between two media of different dielectric constants.

The optical accessory used for studying the polarization of reflected light and the determination of Brewster's angle is shown in figure 1. The accessory consists of green laser pointer, an Industrial Fiber Optics Photometer, rotating table, degree plate, an angular translator component holder, glass or plexiglass plate, optic bench, analyzer arm and a polarizer.

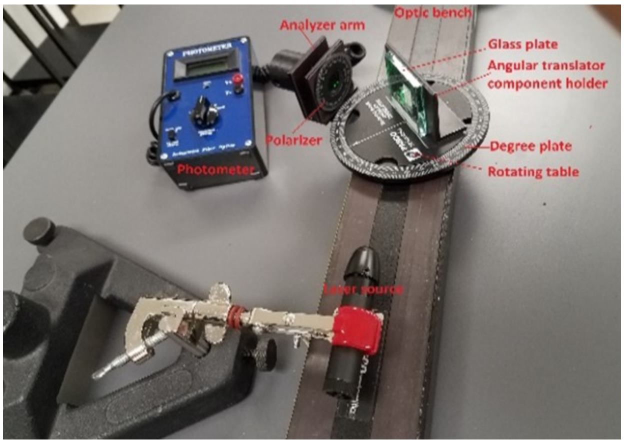

Fig.1: Brewster's angle accessory setup for experiment

When an unpolarized light beam is incident on a flat surface separating two different dielectrics air and glass, it can be decomposed into "s" and "p" components. The plane of vibration

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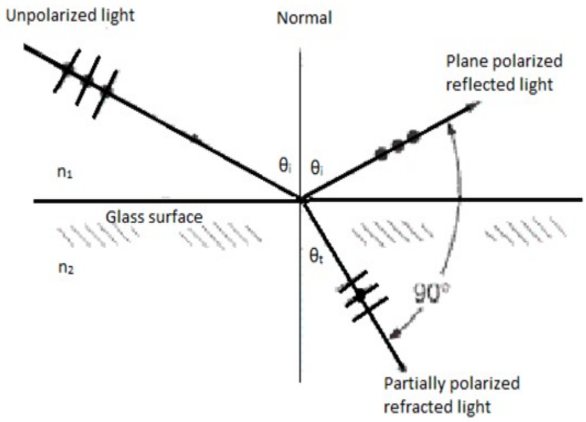
of the "s" and "p" components is perpendicular and parallel to the plane of incidence, respectively. Their intensities depend on the angle of incidence and the refractive indices of materials. At special angle of incidence, the intensity of "p" component of reflected light is zero; and the reflected light becomes plane-polarized with only the "s" component. This particular angle of incidence is called the Brewster's angle or polarization angle [Serway R A and Jewett J W, earth-and-planetary-sciences/brewster-angle]. It was confirmed that the reflected ray and the refracted ray are 90° apart when the incident angle is set at Brewster's angle, as shown in figure2.



I P polarized light, parallel to plane of incidence

• S polarized light, perpendicular to plane of incidence

Brewster's angle of polarization $\theta_i = \theta_B$

Fig.2: Brewster`s angle of polarization

The objective of the of this work was to determined experimentally the component parallel and perpendicular to the plane of incidence of the reflectance and transmittance of an interface and compare them with the theoretical reflectance and transmittance deduced from the Fresnel equations. In addition, from the component parallel to the plane of incidence, also it was possible to determine the Brewster's angle that is the angle at which the reflected intensity is approximately zero. The determination of the Brewster's angle helps to calculate the refractive index of materials.

2. Theory

Brewster's angle

When unpolarized light is incident on flat non-conducting material, such as glass or plexiglass, the reflected light is partially polarized parallel to the plane of the refractive index. The amount of polarization depends on the incident angle and the index of refraction of the reflecting material. there is a specific angle called Brewster's angle at which the light completely polarized. This occurs when the reflected ray and the refracted ray are 90 degrees apart, figure2.

According to Snell's law,

$$n_1\, sin\theta_1 = n_2 sin\theta_2 \quad (1)$$

where n is the index of refraction of the medium and θ is the angle of the ray from the normal.

When the angle of the incident ray is equal to *Brewster's angle*, θ_B ,

$$n_1\, sin\theta_B = n_2 sin\theta_2 \quad (2)$$

and since $\theta_B + \theta_2 = 90^\circ$

$$\theta_2 = 90^\circ - \theta_B, \text{ and}$$

$$sin\theta_2 = \sin(90^\circ - \theta_B) = \sin 90^\circ \cos\theta_B - \cos 90^\circ \sin\theta_B = \cos\theta_B$$

substituting for $\sin\theta_2$ in Eq. (2) gives.

$$n_1\, sin\theta_B = n_2 cos\theta_B$$

Therefore, $\tan\theta_B = \frac{n_2}{n_1} \Rightarrow \theta_B = tan^{-1}\left(\frac{n_2}{n_1}\right)$ (3)

Fresnel equations

The Fresnel equations give the transmission and reflection coefficients at a dielectric interface. They depend upon the polarization and angle of incidence, and indices of refraction of the media on both sides of the interface. For a dielectric medium where Snell's law can be used to relate the incident and transmitted angles, Fresnel's equations can be expressed in terms of incidence and transmission.

Case 1. E perpendicular to the plane of incidence (s component)

The amplitude of reflection and transmission coefficients r_s ($r_\perp$) and t_s ($t_\perp$), respectively, for perpendicular polarized light are.

$$r_\perp = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_1\cos\theta_2} = \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_2+\theta_1)} \quad (4)$$

$$t_\perp = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2} = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)} \quad (5)$$

Case 2. E parallel to the plane of incidence (p component)

The amplitude of reflection and transmission coefficients r_p ($r_\parallel$) and t_p ($t_\parallel$), respectively, for parallel polarized light are.

$$r_{/\!/} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_1\cos\theta_2 + n_2\cos\theta_1} = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)} \quad (6)$$

$$t_{/\!/} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1} = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} \quad (7)$$

The measured reflectance R and transmittance T however are the ratio of flux (not amplitude), such that

$$R = r^2 \quad (8)$$

$$T = \left(\frac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)t^2 \quad (9)$$

$$R + T = 1 \quad (10)$$

$$R_{/\!/} = r_{/\!/}^2 = \frac{tan^2(\theta_1-\theta_2)}{tan^2(\theta_1+\theta_2)} \quad (11)$$

$$R_\perp = r_\perp^2 = \frac{sin^2(\theta_1-\theta_2)}{sin^2(\theta_1+\theta_2)} \quad (12)$$

Hence

$$T_\perp = \left(\frac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)t_\perp^2 = \frac{sin2\theta_1 sin2\theta_2}{sin^2(\theta_1+\theta_2)} \quad (13)$$

$$T_{/\!/} = \left(\frac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)t_{/\!/}^2 = \frac{sin2\theta_1 sin2\theta_2}{sin^2(\theta_1+\theta_2)cos^2(\theta_1-\theta_2)} \quad (14)$$

From Fresnel's equations it can be determined that the parallel reflection coefficient, $r_{/\!/} = 0$, when $\theta_1 + \theta_2 = 90^\circ$ which, indicates the reflected light is polarized parallel to the interface. The use of Snell's law gives an expression for Brewster's angle θ_B , Eq. (3).

If the incident light is unpolarized (containing an equal mix of parallel and perpendicular polarizations) the reflection coefficient is

$$R = \frac{(R_\perp + R\|)}{2} \quad (15)$$

Likewise, the partial polarizations can be written as

$$\frac{R_\|}{R_\perp + R_\|} \quad (16)$$

$$\frac{R_\perp}{R_\perp + R_\|} \quad (17)$$

3. Experimental setup and Procedure

The experimental setup used for this activity is shown in figure3. A green laser pointer that emits a monochromatic beam at 532 nm was adequately aligned in the center of the bench so that the light beam is reasonably parallel with the bench. The angular translator is positioned flush against the alignment rail of the optic bench about 20 cm away from the light source. The degree plate is aligned so that the zero-degree mark is aligned with optical axis of the bench.

The glass plate that will deflect the light is placed in the center of the rotating table that is mounted on the angular translator component holder. The rotating table is then set to various angles, and the angle of incidence is read on the degree plate.

The analyzer arm is rotated to intercept the deflected light. The analyzer arm has built-in component holder a polarizer and a small hole that allows light to get through to the light optic sensor. We have done some modifications in the analyzer arm in order to integrate the casing for light optic sensor of the photometer [Industrial Fiber Optics, Digital photometer].

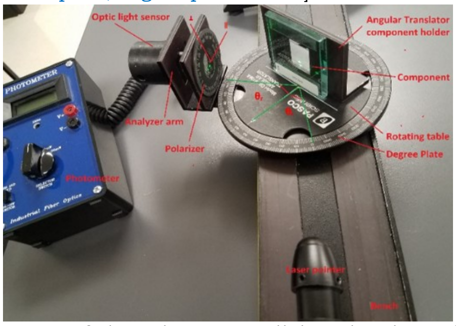


Fig.3: setup of the Brewster's angle of reflected light of polarization

Five trials of each experiment were performed to determine the Brewster's angle of glass plate by varying the angle of incidence light polarized parallel and perpendicular to the plane of incidence in the interval 15°to 85° in 5° increment. The average value of the reflected intensity was then obtained.

Zero the photometer mechanically

The photometer was set to zero mechanically by using a small screwdriver before taking the measurement of the reflected intensities of polarized light parallel and perpendicular to the plane of incidence.

Polarized light intensity Io parallel and perpendicular to the plane of incidence

The transmitted polarized light intensity at the reflector dielectric material, I_0 , at zero degree of normal incidence was measured for parallel and perpendicular to the plane of incidence, respectively.

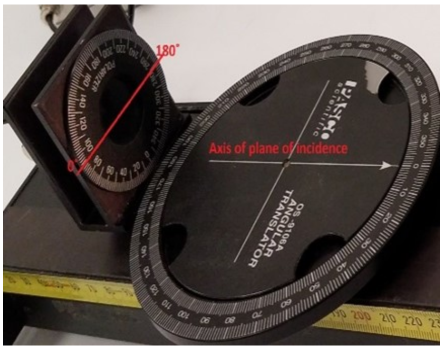
Measurement of reflected intensity parallel I∥ and perpendicular I⊥ to the plane of incidence

The polarizer was fixed in front of the analyzer arm parallel and perpendicular to the plane of incidence, i.e., the 0°-180° axis is horizontal and perpendicular to the axis of light incidence, as shown in figure 4 and 5, respectively. The intensity of reflected light was measured for different angles of incidence between 15° and 85°.

Fig.4: Orientation of the polarizer Parallel to the plane of incidence

Fig.5: Orientation of the polarizer Perpendicular to the plane of incidence

The component holder of glass plate was set at an angle of incidence, θ_i = 15˚, to the axis (0˚-180˚) of plane of light incidence by rotating the angular translator. The degree plate was kept tight in the direction of plane of light incidence, and the analyzer arm



with a polarizer parallel and perpendicular to the plane of incidence was adjusted at a reflected angle $\theta_r = \theta_i$, to measure the reflected light intensity, $I_{\parallel}$ and $I_{\perp}$, respectively. The angle range was limited between 15° and 85° because the expanse of the fiber optic light intensity sensor casing.

The incident angled was then varied between 15° - 85° in 5° increment and the corresponding reflection intensity was measured. It was appeared that the inflection region, which indicates where the Brewster's angle is located, appeared between 54° and 60° where the intensity tends to zero. Hence, measurements of reflected intensity were performed within this range in intervals of 2° to accurately determine where the intensity is a minimum.

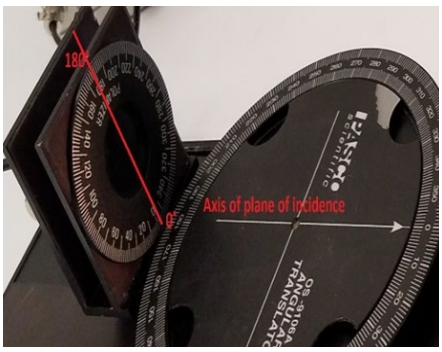
The ratio of reflected intensity which represents the reflectivity of light for parallel and perpendicular to the plane of incidence was then calculated using $R_{II} = \frac{I_{II}}{I_0}$ and $R_{\perp} = \frac{I_{\perp}}{I_0}$, respectively.

The same procedure as mentioned above was repeated for Plexiglas plate. The Fresnel relations were then compared with the experimental data by plotting the predictions made by Eq. (11) and (12) [Lvovky Alexander I 2013].

4. Results and analysis

According to Brewster's law, if the incident light is polarized in the plane of incidence (p-polarization), at some angle of incidence reflectivity is approximately zero R ~ 0. This angle is called the Brewster's angle θ_B and is associated with the refractive index n of the material through the equation (3).

The reflectivity of light intensity for parallel and perpendicular to



the plane of incidence was then calculated using $R_{II} = \frac{I_{II}}{I_0}$ and $R_{\perp} = \frac{I_{\perp}}{I_0}$, respectively. The graphs of reflectivity ($\frac{I_{II}}{I_0}$ and $\frac{I_{\perp}}{I_0}$) along with R = (R⊥ + R∥)/2 as a function of angle of light incidence θi for P and S components were plotted for glass and plexiglass, as shown in figure 6 and 7, respectively.

As can be seen from data points of these plots, the reflectivity of parallel and perpendicular polarized components increases with varying angle of incidence θi from approximately 0.048 to 0.52 and 0.055 to 0.55 for glass and plexiglass, respectively. The shapes of P and S polarized components are different from each other. The reflectivity of polarized P-component for both materials decreases smoothly by varying angle of incidence from 15° to the inflection region between 54°- 60°, then starts to increase abruptly from that region to 85° to reach the maximum of reflectivity that corresponds to approximately 0.52 and 0.55 for glass and plexiglass, respectively.

The lowest points of inflection for several trials were (57±0.5)° and (56±0.5)°. These values correspond to the Brewster's angle θB for glass and Plexiglass, respectively. At these points, the reflectivity is approximately zero. At the same time, the reflectivity of polarized S-component increases gradually with varying angle of light incidence from 15° to 85°, from 0.059 to attain the maximum of reflectivity to approximately 0.75 for both dielectric materials.

These plots also show an angle where the horizontal polarization exactly vanishes. This angle represents the Brewster's angle, θB, its value for glass-air and plexiglass-air interface is approximately (57±0.5)° and (56±0.5)°, respectively. At this angle the reflected light is completely polarized.

Figures 8 and 9 show the plots of reflectivity of light intensity of polarized light P and S components as a function of angle of incidence, with predictions made by the Fresnel relations, Eq. (11) and (12), for glass and plexiglass dielectric materials, respectively.

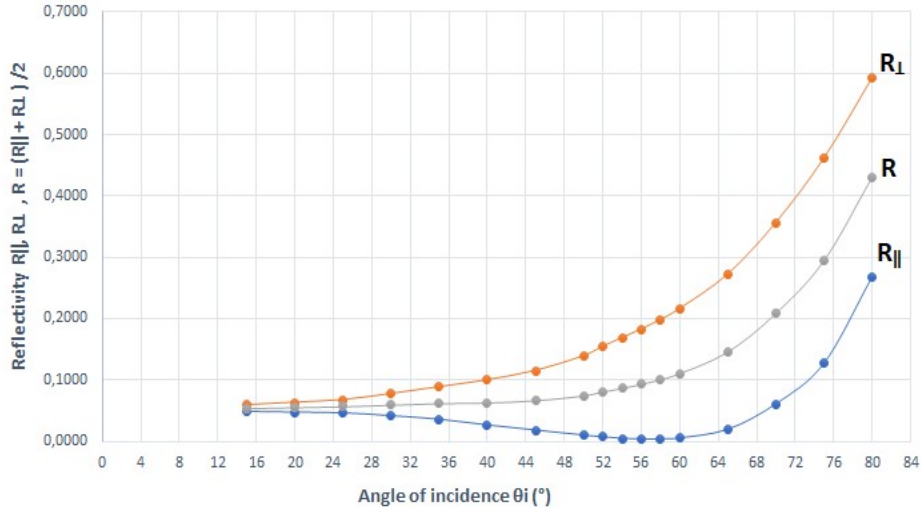


Fig.6. Angular dependence of the reflection of P and S -polarized light for glass plate material. Bleu dotted line: P-component; Red dotted line: S-component.

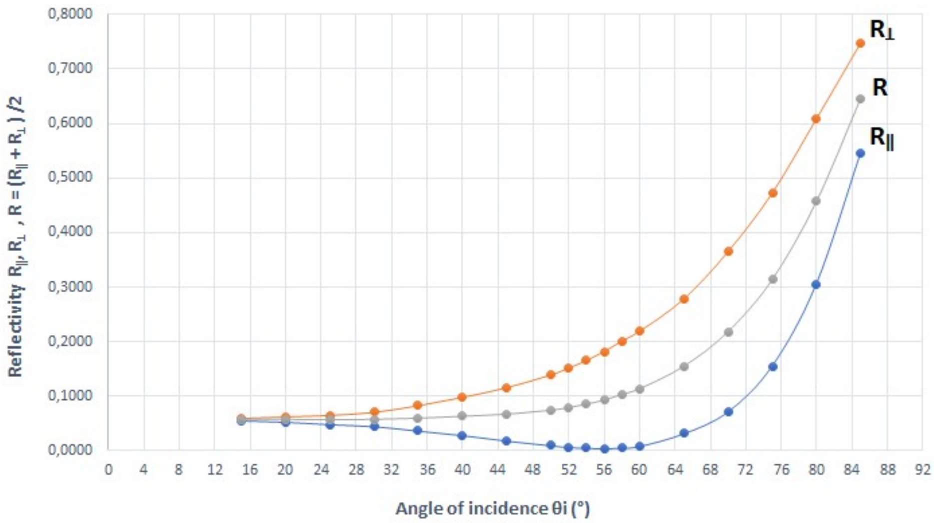


Fig.7. Angular dependence of the reflection of P and S -polarized light for plexiglass plate material. Bleu dotted line: P-component; Red dotted line: S-component.

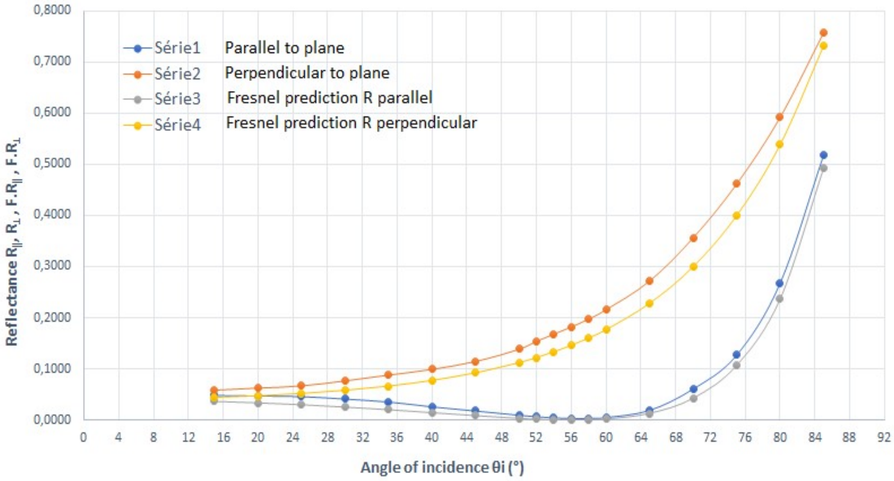


Fig.8. Reflected intensity of light versus variable angles of light polarized parallel and perpendicular to the plane of interface through a glass slide, together with the Fresnel predicted R parallel and R perpendicular curves

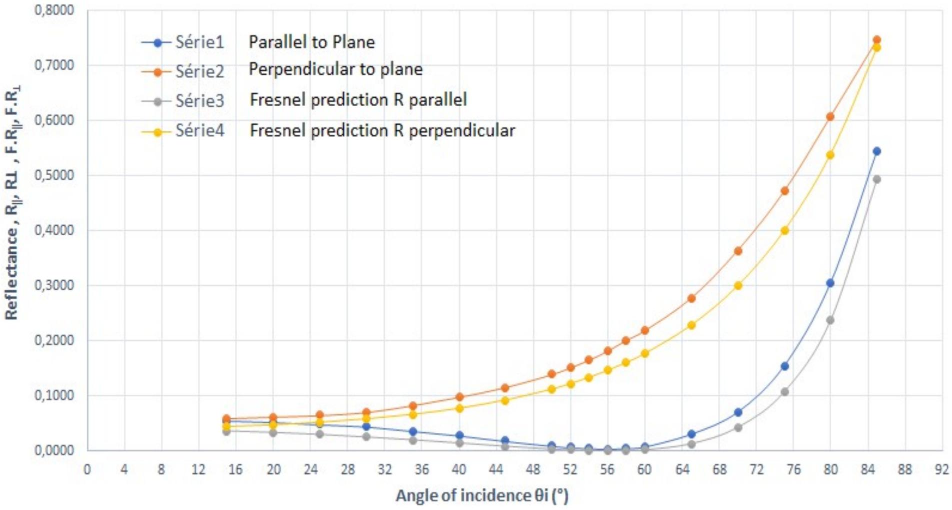


Fig.9. Reflected intensity of light versus variable angles of light polarize parallel and perpendicular to the plane of interface through a plexiglass slide,together with the Fresnel predicted R parallel and R perpendicular curves

As it can be seen from the plots of figures 8 and 9 that the data obtained for reflection coefficient with varying incident angle for both polarization orientations are in good agreement with the theoretical predictions made by Fresnel equations. The reflectance for parallel polarization goes to zero around Brewster's angle whilst the intensity for perpendicular polarization continues to rise as incident angle increases. The values for R‖ are much more consistent with the theoretical predictions compared with R⊥, expect at incident angles between (15-30) ° and 85° where it is evident that the values for R⊥ are closer to the predictions made by Fresnel equations compared with R‖.

The value of refractive index was then determined using Eq. (3) and the theoretical refractive index of air nair =1.0. Therefore, the obtained average value of nglass and nPlexiglas were 1.539 ±0.006 and 1.4825±0.008 for glass and Plexiglas, respectively. The result of both dielectric materials is in good agreement with the accepted value 1.52 and 1.495 for glass and Plexiglass, respectively, [https://en.wikipedia.org/wiki/List_of_refractive_indices, https://physics.info/refraction/, https://robinwood.com/Catalog/Technical/Gen3DTuts/Gen3DPages/RefractionIndexList.htm]. The percentage difference between the obtained result and the accepted value for glass and plexiglass was 1.2 % and 0.87 %, respectively.

5. Conclusion

The values of Brewster's angle for glass and Plexiglas were determined from the graph reflectance versus variable angles of light polarized parallel and perpendicular to the plane of interface through dielectric materials, together with the Fresnel predicted R parallel and R perpendicular curves. The values of the Brewster's angle were found to be (57±0.5) ° and (56±0.5) ° for glass and plexiglas, respectively. These results were in accordance with what was expected. Using the Brewster's Law with the index of refraction of air n1 = 1.0, the weighted mean calculation of the refractive index was found to be nglass = 1.539±0.006 and nPlexiglas = 1.482±0.008 for glass and Plexiglas, respectively. Both values are matching the accepted values as they are both within an error of approximately 1 % of the values given by [7, 8, 9].

The percent error between Brewster's angle for glass and Plexiglas was approximately 1.7% which represents roughly 1° of difference. From the percentage difference of the two Brewster's angle, we can say that there is no evidence of the material identity can be determined from the value of the Brewster's angle since the Brewster's angle is almost

the same for glass and Plexiglas and in some cases their values overlap, depending upon their chemical compositions.
This work can be a subject of laboratory experiment for college and under graduate students in physics. It can be also extended as a research item to investigate how the refractive index of materials is dependent on wavelength of light incident on it, by considering the effects of dispersion.

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